Re-exam Waves and Optics – 27 February 2014

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This exam contains 6 questions on 5 pages.

A few **preliminary remarks**:

- Answers may be given in Dutch.
- Use a new sheet of paper for each question. For questions you do not answer, please hand in a blank sheet to make this clear.
- Put your name and student number at the top of all sheets.
- Put your student card at the edge of the desk for checking by the assistants and show it when handing in your exam.

Question 1 (6 points): radiation pressure

For light impinging on a **perfectly absorbing surface**, the time-averaged radiation pressure P exerted by the light is given by: $P = \frac{I}{c}$, with I the irradiance of the light and c the speed of light (3×10^8 m/s).

Now consider a light beam with an irradiance of 5.00×10^6 W/m^2 impinging normally on a surface that reflects 40% and absorbs 60% of the irradiance.

Question:

Calculate the resulting time-averaged radiation pressure on the surface. Give the physical units for all calculated quantities.

(Question 2 on the next page)

Question 2 (5 points): The evanescent wave

The figure below sketches the situation of internal reflection. When the angle of incidence θ_i equals the critical angle, the angle of refraction θ_t equals 90 degrees. For angles of incidence θ_i larger than the critical angle, total internal reflection occurs (there is no refracted light ray). However, in this case, a special wave, the so-called evanescent wave, runs along the interface (along the x-axis in the figure). The evanescant wave can be expressed as:

$$\overline{E}_t = \overline{E}_{0t}\, e^{-\beta y}\, e^{i\left(k_t x \frac{n_i}{n_t} \sin\theta_i - \omega t\right)}$$

with n_i, n_t the indices of refraction of the two media. The parameter β scales as $1/\lambda$, with λ the wavelength of the light.

Questions:

a) How can the evanescent wave be used to construct a beam splitter, a device that splits a beam of light in two parts ? Which factor of the expression for the evanescent wave given above is responsible for this ?

b) How can one in such a device control/change the relative amounts of light that go into the two beams ?

Hint: This use of the evanescent wave was demonstrated during the lectures making use of microwaves and parafin blocks.

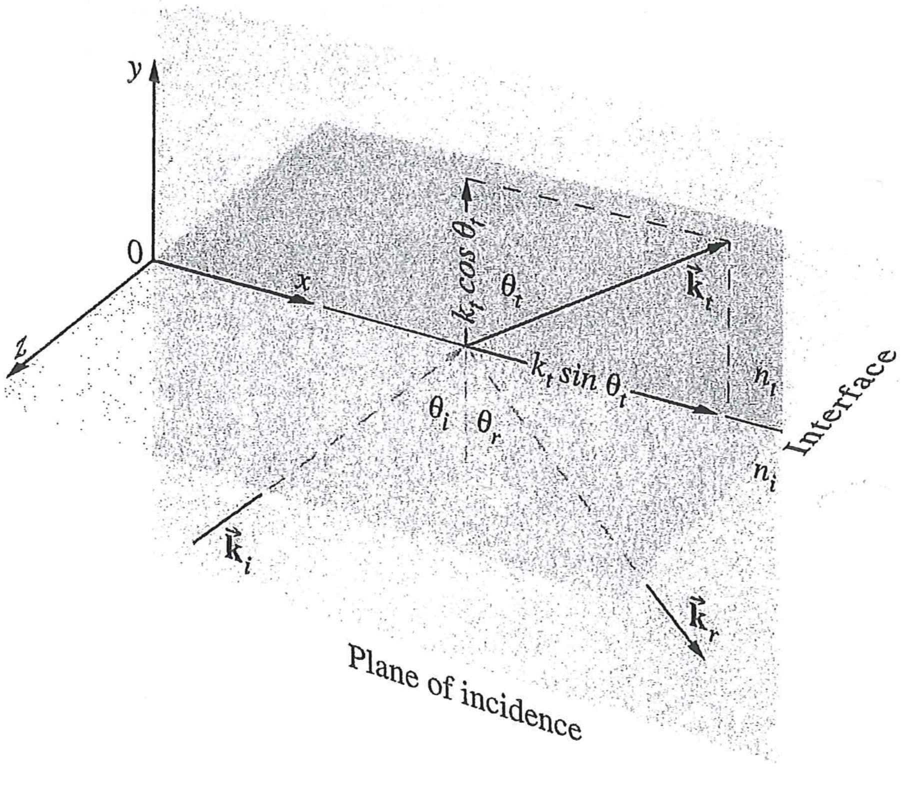


Figure 4.54 Propagation vectors for internal reflection.

(Question 3 on the next page)

Question 3 (6 points): Standing waves

The following expression represents a standing wave:

$$E = 100\ \sin\tfrac{2}{3}\pi x\ \cos 5\pi t$$

(x is the spatial coordinate, t is time)

Questions:

a) Why is this called a "standing" wave ?

b) Give 2 wave functions which, when superposed, generate the standing wave given above.

Question 4 (8 points): linear polarization

Write an expression for the space and time dependence of the electric field of a linearly polarized electromagnetic wave with the following properties:

- angular frequency ω
- amplitude E_0
- propagation along the positive x-axis
- propagation number k
- the plane of vibration has an angle of 25 degrees with the xy-plane
- the size of the electric field at $x = 0$, $t = 0$ is equal to E_0

Include a sketch of the situation showing the amplitude vector and the direction of propagation in a cartesian (rectangular) coordinate system.

Hint: write the amplitude vector in terms of the unit vectors along the axes of the coordinate system.

(Question 5 on the next page)

Question 5 (9 points): Young's experiment

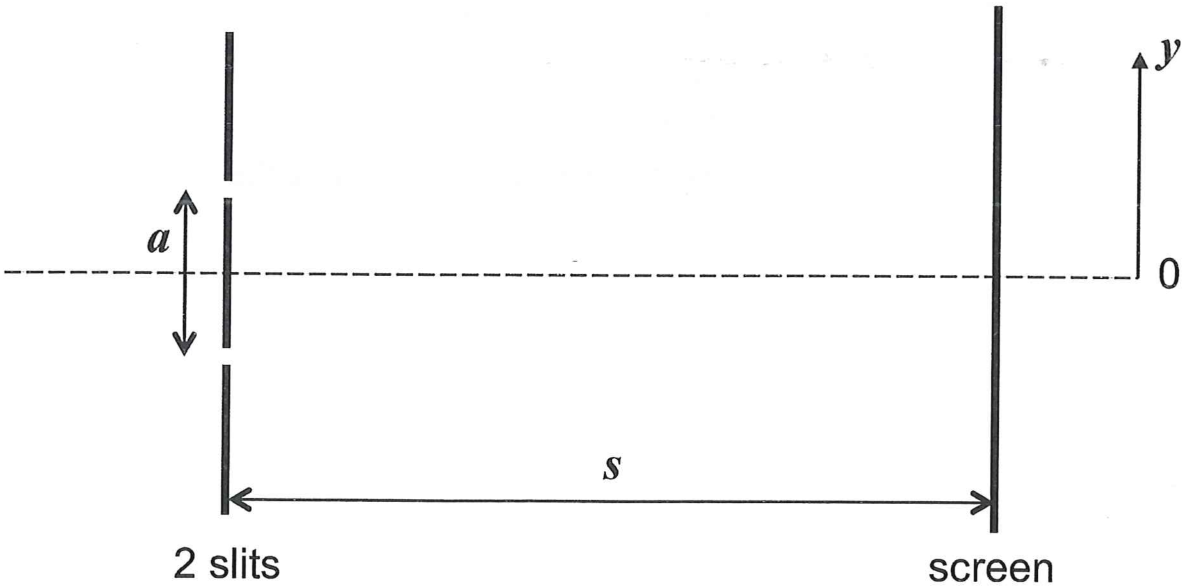
Young's experiment considers the interference of coherent light passing through 2 slits of negligeable width. An interference pattern consisting of bright and dark lines parallel to the slits is observed on a screen.

Making use of some appropriate approximations, derive that the distance between two consecutive maxima (the bright lines) in the interference pattern is given by:

$\Delta y_{\text{max}} \approx \frac{s}{a}\lambda$, with

- s: distance from the slits to the screen;
- a: distance between the 2 slits;
- y: position coordinate in the screen along the direction perpendicular to the length of the slits;
- λ: wavelength of the light.

Make a drawing of the situation to illustrate your answer. The drawing below helps to sketch the situation of Young's experiment and can be used as starting point to illustrate your answer.



Sketch of the situation of Young's experiment.

(Question 6 on the next page)

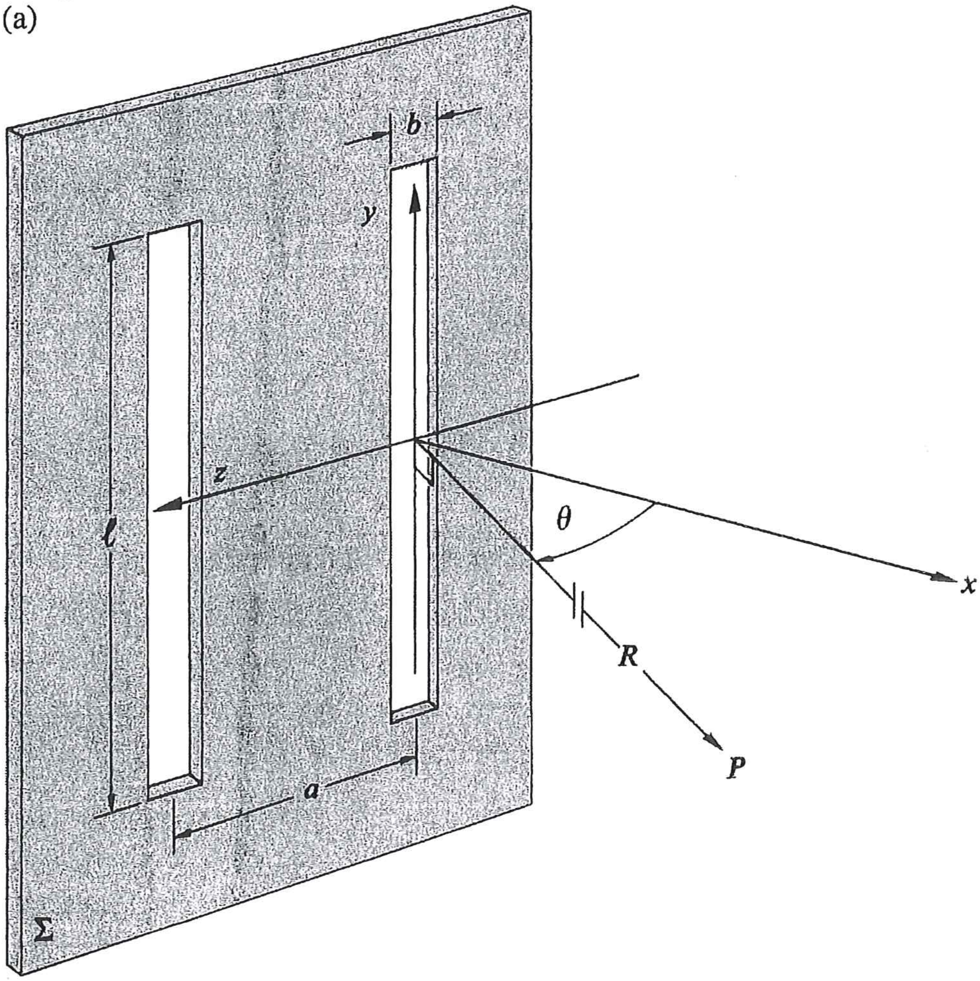
Question 6 (6 points): double-slit diffraction

The Fraunhofer diffraction pattern (irradiance) of a double slit is given by:

$$I(\theta) = 4I_0\left(\frac{\sin\beta}{\beta}\right)^2 \cos^2\alpha, \quad \text{with} \quad \beta = \frac{\pi\, b\sin\theta}{\lambda}, \; \alpha = \frac{\pi\, a\sin\theta}{\lambda}$$

and I_0 the irradiance for $\theta = 0$ for a single slit. The formula contains a factor representing the diffraction of a single slit and a factor representing the interference of two line sources. The following drawing (fig. 10.13 from Hecht) illustrates the parameters and variables used. P represents a point on the screen where the diffraction pattern is observed.

(a)



Consider a double slit with slit widths $b = 0.05$ mm for which the fourth-order interference maxima are missing from the diffraction pattern (a so-called missing order).

Note: the **zeroth-order maximum is the central maximum**: $I(\theta = 0)$.

Question:

How big is the slit separation a ?